EJERCICIOS DE ECUACIONES NO EXACTAS

1. (y^3 +2e^xy) dx + (e^x+3y^2)dy=0

Solución

$\frac{\partial M}{\partial y} = 3y^2 + 2e^x$ $\frac{\partial N}{\partial x} = e^x + 3\, y^2$ $\frac{\partial N}{\partial x} = e^x$

Ecuación no exacta

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$$

$$= \frac{3y^2+2e^x - e^x}{e^x+3y^2}$$

$$= \frac{3y^2 + e^x}{e^x+ 3y^2}$$

Cancelamos

p(x)=1

p(x)=1 $e^{\int 1dx} = e^x$

e^x (y^3+2e^xy) dx+ e^x (e^x+3y^2) dy=0

($e^x y^3$+2e^{2x}y) dx + (e^{2x}+3 $e^x y^2$) dy=0

$\frac{\partial M}{\partial y}$ =3$e^x y^2$+2e^{2x} $\frac{\partial N}{\partial x}$ = 2e^{2x}+ 3$e^x y^2$

Ecuaciones exactas

$\frac{\partial f}{\partial x}$ = e^x y^3+ 2e^{2x}y $\frac{\partial f}{\partial y}$=$e^{2x}$+3 $e^x y^2$

F(x, y)=$\int M(x,y) + g'(y)$

F(x, y)=$\int \mathrm{e^x y^3} + 2\mathrm{e^{2x} y} + g'(y)$

Integramos con respecto a x

F(x, y)= $\frac{e^{x}y^{\wedge}4}{4}$ + $\frac{2e^{2x}y^{\wedge}2}{2}$ + $g'(y)$

= $\frac{e^{x}y^{\wedge}4}{4}$ ' $e^{2x}y^{\wedge}2$ – + $g'(y)$

$\frac{\partial f}{dy}$= e^{2x}+3 $e^{x}y^{2}$

=2 e^{2x}+ 3 $e^{x}y^{2}$ + $g'(y)$

2 e^{2x}+ 3 $e^{x}y^{2}$ + $g'(y)$ = e^{2x}+3 e^{x} y^{2}

$g'(y)$ = -2e^{2x}-3 $e^{x}y^{2}$+ e^{2x}+3 $e^{x}y^{2}$

=-1

Solución

$\frac{e^{x}y^{4}}{4}$ ·· $e^{2x}y^{2}$ – $+ (-1)$

$\frac{e^{x}y^{4}}{4}$ + $e^{2x}y^{2}$ -1 +c

2. (3 xy +y^{2}) dx + $(x^{2} + xy)$ $dy = 0$

$M = (3\,xy\ + y^{\wedge}2)$ $N = (x^{2} + xy)$

$\frac{\partial M}{\partial y} = 3x + 2y$ $\frac{\partial N}{\partial x} = 2x + y$

Aplicamos la siguiente formula:

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{3x + 2xy - (2x + y)}{x^{2} + xy}$$

Operamos signos

$$= \frac{3x + 2y - 2x - y}{x^{2} + xy}$$

Cancelamos términos semejantes

$$= \frac{x+y}{x^2+xy}$$

$$= \frac{x+y}{x(x+y)}$$

Obtenemos nuestra $p(x)$

$p(x) = \frac{1}{x}$

$p(x) = \int \frac{1}{x} dx$

$= \int \frac{dx}{x}$

$= \ln x$

El resultado de la integral debe ser multiplicado por cada lado.

$x(3\,xy + y^2)\,dx + x\,(x^2 + xy)\,dy = 0$

$(3\,x^2y + xy^2)\,dx + (x^3 + x^2y)\,dy = 0$

Verificamos los términos de M y N

$M = (3\,x^2y + xy^2)$

$N = (x^3 + x^2y)$

Pasamos a derivar

$\frac{\partial M}{\partial y} = 3x^2 + 2xy$ $\frac{\partial N}{\partial x} = 3x^2 + 2xy$

Ecuaciones exactas

$\frac{df}{dx} = 3x^2y + xy^2$ $\frac{df}{dy} = x^3 + x^2y$

Tenemos en cuenta la siguiente formula.

F(x, y)=$\int M(x,y) + g'(y)$

F(x, y)=$\int M(3x^2y + xy^2) + g'(y)$

Integramos

F(x, y)= $\frac{3x^3y}{3} + \frac{x^2y^2}{2} + g'(y)$

F(x, y)= $x^3y + \frac{x^2y^2}{2} + g'(y)$

Derivamos con respecto a y

F(x, y)= $x^3 + \frac{2x^2y}{2} + g'(y)$

$= x^3 + x^2y + g'(y)$

Reemplazamos e igualamos a N

$x^3 + x^2y + g'(y) = x^3 + x^2y$

Despejamos $g'(y)$

$g(y) = -x^3 - x^2y + x^3 + x^2y$

Operamos

$g(y) = 0$

Integramos

$\int g'(y) = \int 0$

$g(y) = 0$

Solución

$\frac{x^3y}{} + \frac{x^2y^2}{2} + 0 + c$

$= \frac{x^3y}{} + \frac{x^2y^2}{2} + c$

ECUACIONES EXACTAS

1. $(2xy^2+1)dx+2x^2y\,dy=0$

$M(x,y)=2xy^2+1$

$N(x,y)=\ 2x^2y$

$$\left.\begin{aligned}\frac{\partial M}{\partial y}&=2x2y=4xy\\[2em]\frac{\partial N}{\partial x}&=2y2x=4xy\end{aligned}\right\}\ \text{Ecuaciones exactas}$$

Tenemos que las derivadas de las funciones son las siguientes:

$$\frac{df}{dx}=\underbrace{2xy^2+1}_{M}\qquad\qquad \frac{df}{dy}=\underbrace{2x^2y}_{N}$$

Aplicamos la formula

$f(x,y)=\int M(x,y)+g'(y)$

Reemplazamos M:

$f(x,y)=\int M(2xy^2+1)+g'(y)$

Integramos con respecto a x

$f(x,y)=\frac{2x^2y^2}{2}+x+g'(y)$

$f(x,y)=\frac{\cancel{2}x^2y^2}{\cancel{2}}+x+g'(y)$

$f(x,y)=x^2y^2+x+g'(y)$

Tenemos la derivada de la función con respecto a y

$\frac{df}{dy}=2x^2y+g'(y)$

$2x^2y+g'(y)=2x^2y$

$g'(y)=2x^2y-2x^2y$

$$g'(y) = 0$$

$$g(y) = 0$$

$$f(x, y) = x^2y^2 + x$$

La solución de la ED

$$x^2y^2 + x + c = 0$$

$$y^2 = \frac{-c - x}{x^2}$$

$$\mathrm{y} = = \frac{\sqrt{-c - x}}{x}$$

2. $(2x - 1)dx + (3y + 7)dy = 0$

$$M(x, y) = 2x - 1$$

$$N(x, y) = 3y + 7$$

La derivada de la función con respecto a la variable que esta como denominador.

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial M}{\partial x} = 0$$

Ecuaciones exactas

$$\frac{df}{dx} = 2x - 1 \qquad \frac{df}{dy} = 3y + 7$$

$$f(x, y) = \int M(x, y) + g'(y)$$

$$f(x, y) = \int (2x - 1) + g'(y)$$

Integramos con respecto a x

$$f(x, y) = \frac{\cancel{2}x^2}{\cancel{2}} - x + g'(y) \quad \Rightarrow \quad f(x, y) = x^2 - x + g'(y)$$

$$\frac{df}{dy} = 3y + 7 + g'(y)$$

$$3y + 7 + g'(y) = 3y + 7$$

$g'(y) = -3y - 7 + 3y + 7$

$g'(y) = 0$

$f(x,y) = x^2 - x + 0$

La solución de ED: $x^2 - x + 0 + c = 0$

: $x^2 - x + c$

3. $2xy\,dx + (x^2 - 1)\,dy = 0$

   $M(x,y) = 2xy$
   $N(x,y) = x^2 - 1$

   Pasamos a derivar

$\frac{\partial M}{\partial y} = 2x$ $\frac{\partial N}{\partial x} = 2x$

Ecuaciones exactas

$\frac{\partial f}{\partial x} = 2xy$ $\frac{\partial f}{\partial y} = x^2 - 1$

$f(x,y) = \int M(x,y) + g'(y)$

Reemplazamos

$f(x,y) = \int 2xy + g'(y)$

Integramos con respecto a x

$f(x,y) = \frac{2x^2y}{2} + g'(y)$

Cancelamos

$f(x,y) = \frac{\cancel{2}x^2y}{\cancel{2}} + g'(y)$

$f(x,y) = x^2y + g'(y)$

Tenemos la derivada

$\frac{\partial f}{\partial y} = x^2 - 1$

$g'(y) = -1$

Luego integramos

$g(y) = -\int dy = -y$

La solución general

$x^2y + g'(y)$
$x^2y + (-y)$
$la\ solucion\ de\ la\ ED\ x^2y - y + c$

4. $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$

$M(x, y) = 2xy - \sec^2 x$

$N(x, y) = x^2 + 2y$

Pasamos a derivar

$\frac{\partial M}{\partial y} = 2x$ $\frac{\partial N}{\partial x} = 2x$

Ecuaciones exactas

$\frac{\partial f}{\partial x} = 2xy - \sec^2 x$ $\frac{\partial f}{\partial y} = x^2 + 2y$

$f(x, y) = \int M(x, y) + g'(y)$

Reemplazamos

$f(x, y) = \int 2xy - \sec^2 x + g'(y)$

Integramos con respecto a x

$f(x, y) = \frac{\cancel{2}x^2y}{\cancel{2}} - \tan x + g'(y)$

Cancelamos

$f(x, y) = x^2y - \tan x + g'(y)$

Tenemos la derivada

$\frac{\partial f}{\partial y} = x^2 + 2y$

$g'(y) = 2$

Luego integramos

$g(y) = \int 2dy = y^2$

La solución

$x^2y - \tan x + y^2 = 0$

$x^2y - tanx + y^2 + c$

5. $(3x^2 - y)dx + (3y^2 - x)dy = 0$

$M(x,y) = 3\mathrm{x}^2 - \mathrm{y}$

$N(x,y) = 3y^2 - x$

Pasamos a derivar

$\frac{\partial M}{\partial y} = -1$ $\frac{\partial N}{\partial x} = -1$

Ecuaciones exactas

$\frac{\partial f}{\partial x} = 3x^2 - y$ $\frac{\partial f}{\partial y} = 3y^2 - x$

$f(x,y) = \int M(x,y) + g'(y)$

$f(x,y) = \int (3x^2 - y) + g'(y)$

$f(x,y) = \frac{\cancel{3}x^3}{\cancel{3}} - xy + g'(y)$

$f(x,y) = x^3 - xy + g'(y)$

$\frac{\partial f}{\partial y} = 3y^2 - x$

$g'(y) = 6y$

$g(y) = \int 6dy = \frac{6y^3}{3} = y^3$

$f(x, y) = x^3 - xy + \mathrm{y}^3$

La solución de la ED $x^3 - xy + \mathrm{y}^3 + c$